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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

APPLICATION OF THE SONAR EQUATIONS TO
BISTATIC ECHO-RANGING

by

Lawrence Michael Harvey

March 1982

Thesis Advisor: A. B. Coppens

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The thesis explores the phenomena unique to echo-ranging with a source widely separated from the receiver. In an asset-austere era of antisubmarine warfare, this technique serves as a tactical advantage, particularly in the passive tracking of a submarine. Particular emphasis is placed on the terms of the sonar equation most affected by the bistatic

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Application of the Sonar Equations to
Bistatic Echo-Ranging

by

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Lieutenant, United States Navy
B.S., United States Naval Academy, 1974

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN SYSTEMS TECHNOLOGY

from the

NAVAL POSTGRADUATE SCHOOL
March 1982

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ABSTRACT

The thesis explores the phenomena unique to echo-ranging with a source widely separated from the receiver. In an asset-austere era of antisubmarine warfare, this technique serves as a tactical advantage, particularly in the passive tracking of a submarine. Particular emphasis is placed on the terms of the sonar equation most affected by the bistatic geometry: Reverberation level and target strength. The research is particularly applicable to ongoing NATO and naval laboratory work involving the bistatic concept in array design and for use with surface escorts in conjunction with friendly submarines.

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I. INTRODUCTION

The twentieth century heralded the employment of a type of warfare only dreamed of or lightly tcyed with in the previous centuries of human conflict. The instrument of this new development, undersea warfare, was the submarine and it brought concern about the underwater world quickly and dramatically to military strategists and researchers alike. In its infancy, the submarine was an adjunct of the surface fleet; it operated primarily on the surface and utilized many surface tactics and used its capability to submerge only when stalking victims or retreating from attackers. The advent of radar and sea-based aircraft diminished the capability of the submarine to operate successfully on the surface for any length of time.

The next technological innovations were on the side of the submarine; snorkels, periscopes, and submerged launch weapons which allowed for submerged transit and attack while offering only a minimal surface area for counterdetection. Further improvements in propulsion systems and batteries improved these capabilities of submerged operations. The submarine had become sufficiently sophisticated that its reason for existence was in its ability to operate below the sea surface in an environment which effectively denied the use of electromagnetic waves in its detection. The tactical advantage of the submarine therefore, was its ability to conduct all phases of attack, evasion, and retreat without being observed.

This was the problem faced by those forces charged with the detection of the submarine and the denial of its mission; the submarine must be detected in its primary operating medium, submerged in the ocean. To develop the most effective means of detecting submerged targets, various observational agents were evaluated with respect to range and velocity of penetration and ability to distinguish or resolve one target from another. Electromagnetic radiation, which had proved so successful in the atmosphere, was found to be effective only to short ranges because of the extreme attenuation and scattering encountered. Optical systems and "underwater radar" were thus eliminated. Other potential detection techniques based on magnetic field generation or perturbation, electrical field generation, or hydrodynamic effects, to name but a few, were either technologically unfeasible, or offered short detection ranges, or else provided unreliable detection.

The system which provided the best results with respect to the criteria of evaluation was one that utilized sound as the agent of detection. Acoustic detection systems, called passive sonars, were developed that were capable of exploiting sound generated by the target. Other systems generated bursts of acoustic energy in the seawater and collected echoes of returning energy "bounced" from the target. These were called active sonars.

The conflict between those who operate submarines and those who seek them has continued the game of technological

"leapfrog" initiated by the first operational submarine. Each advance in a particular system leads to renewed efforts in the counter-system to deny any advantage to the advance. State-of-the-art technology finds submarines operating more quietly, because of internal quieting and because both nuclear and improved diesel-electric propulsion systems are quieter; operating submerged longer for the same reasons; using longer range weapons which may be targeted and launched without endangering the submarine by bringing it into counter-detection range; and using sophisticated counter-measures and sound absorption and controlled reflection techniques. Antisubmarine forces employ computer-based signal processing, more effective passive sonars, air-deployed sonobuoys, improved coordinated tactics, and advanced array designs to diminish the submarines advantage and possibly establish their own. It appears, however, at least on an operational basis, that the submarine currently possesses the upper hand in accomplishing its assigned mission. It is possible that the capabilities of the antisubmarine equipment and operators are nearing the current technological limits of acoustic systems. It is also possible that the next advance against the submarine will involve a novel employment of existing systems, in a force multiplier role, to regain tactical advantage. Bistatic sonar techniques, employing characteristics of both passive and active sonar systems, may provide one such improvement in capability.

A. PHYSICAL DESCRIPTION

The primary objectives of an underwater surveillance system are the detection, clasification, and tracking of submerged targets by listening for target-generated noise or by echo-ranging. Modern operational systems employ passive sonar to accomplish listening objectives and strictly monostatic active sonar to perform echo-ranging. Passive sonar is used to detect sounds generated by the submarine such as propulsion noise, flow noise, and cavitation. The transmission of this sound is in one direction only, from target to receiver. This sound will provide an operator with an accurate target bearing only; range determination, although implemented, is more complicated and not always instantaneous. These sounds also have components spread over a wide range of frequencies. The advantage of a passive system is that the listener remains undetected by the target, or in the worst case, the target does not feel that the listener is alerted to its presence. However, passive systems have several drawbacks. The operator must be able to distinguish a target signal that varies little from the background, and must be familiar with both target sounds and various background noise. He must also be able to distinguish between the various sounds indicative of different ship operating conditions in order to classify the source. Passive systems are also more easily decoyed.

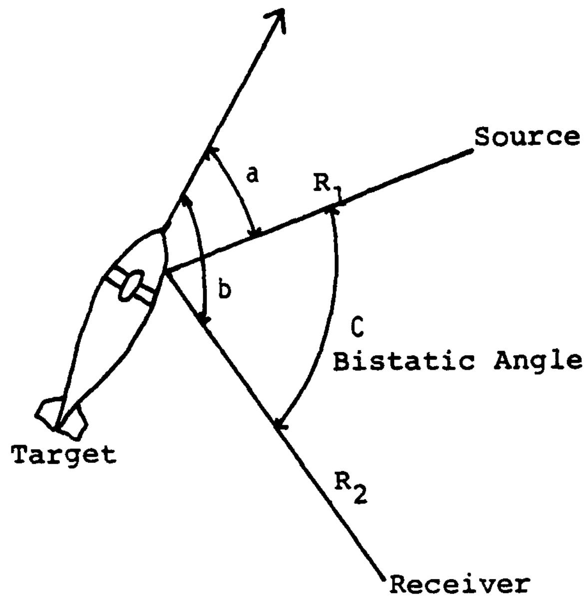
Echo-ranging employs the transmission of a powerful acoustic pulse, or sonar signal, which, it is hoped, will strike a target. The target reradiates the incident sound energy, acting like a secondary source, in all directions including that of the transmitter. The transmitter itself or a nearly contiguous receiver detects the returning sound, or echo, and converts it to a usable presentation. The time interval between the transmission of the signal and the detection of the echo combined with the speed of sound in the water results in the target range:

$$\text{Range} = \frac{(\text{speed of sound}) \times (\text{time})}{2}$$

Echo-ranging is thereby utilized to: (1) establish contact acoustically with a target, (2) maintain contact and classify the target, which includes target range and bearing, and (3) develop range rate and bearing rate of change of the target. Echo-ranging is dependent on the presence and recognition of an echo from a target: the target must be ensonified, the sound energy must return to the transmitting source, and the echo must be of a quality and relative strength to enable processing and perception by the operator. Factors involved in echo-ranging that are not relevant to passive listening include: (1) two-way transmission loss, from source to target and from target to receiver; (2) source level of acoustic power transmitted; (3) target strength or reradiation characteristic of the target; (4) effects of frequency

selection; high frequencies improve resolution but yield shorter ranges due to higher attenuation; (5) effects of doppler; and (6) reverberation resulting from ensonification of scatterers in the vicinity of the target including the sea surface and bottom. The primary non-physical difference between passive and active sonar is in tactical employment. Both may be used offensively or defensively but active sonar is often used by surface vessels which cannot conceal their position due to the high level of noise they generate. Submarines, on the other hand, try to engage offensively as covertly as possible and therefore rely on passive sonar with echo-ranging used sparingly. Recent trends in surface ship quieting have resulted in more widespread use of passive tactics similar to those of the submarine by the surface community.

A bistatic sonar system has some of the characteristics and qualities of both passive and echo-ranging sonars. In a bistatic case, the source and receiver are physically separated by an appreciable distance (Figure 1). The source echo-ranges, ensonifying the water. If a target is present, it radiates incident acoustic energy as in the monostatic echo-ranging case. In this manner, the target can be considered a source, though technically a secondary source, in a passive engagement. The reradiated sound energy, besides returning toward the transmitter (although not necessarily of a sufficient signal-to-noise ratio to allow detection at the



a - Aspect with Respect to Source

b - Aspect with Respect to Receiver

R_1 - Range from Source to Target

R_2 - Range from Target to Receiver

C - Bistatic Angle

Figure 1. Bistatic Geometry

transmitter), also travels in the direction of the displaced receiver. The sound experiences transmission loss over two paths as in the monostatic case but the lengths of the paths and the attenuation factors involved may not be the same. The receiver derives the target bearing from the received echo and, through the application of geometry involving the position of the source relative to the receiver and the distances implied by the travel time of the acoustic signal, the target range can also be computed. As in listening, the position of the receiver is not easily detected by the target and all of the products of echo-ranging, contact, classification, and rates, are available to the operator (though not instantaneously). The echo is in a smaller frequency range for detection but the operator must still be able to distinguish the echo from the background, and particularly from the transmitted acoustic pulse that reaches his position directly. Source level, frequency selection, doppler shift and reverberation are still important factors in the bistatic case. The receiver must operate in the frequency range of the source or in the range of a harmonic of the source signal and often, the receiver will be a sonar similar to the source but employed in the passive mode. The monostatic echo-ranging capability of the source is not diminished. Also, energy that, in the monostatic case, was useful only to the target for counterdetection of the source, can be used for target surveillance through resourceful placement of the bistatic receiver.

B. ADVANTAGES AND DISADVANTAGES OF BISTATIC SYSTEMS

In listing the advantages and disadvantages of a particular system or technique, it is important to limit the comparisons to those aspects that are pertinent to the actual intended application of the system. It is therefore essential to define the system's proposed application including usage and desired results. Earlier experimentation with bistatic systems was centered around developing a means of increasing the area of coverage available to the system user. While this is still a worthwhile goal, the bistatic coverage area may, under certain geometries, be mathematically restricted to an area less than that achievable via monostatic sonar employment.

Bistatic sonar is considered in this development as a possible means of force multiplication in an asset-austere era of antisubmarine warfare. The importance of bistatics will be measured by its ability to provide a tactical advantage, and it is in this light that relative advantages and disadvantages of such a system are presented. In the ultimate, the tactical applications are the most important: the use of bistatics to not alert the target and to put a weapon on the target.

The advantages of a bistatic system result from the tactics against which it will be employed. These advantages include: (1) Passive tracking. The submarine can be tracked by the receiving unit to the extent that a fire control

solution may be generated. The submarine may be aware of the active source but may feel unthreatened because of the source's long range or apparent lack of response to a potential contact. This may, in fact, be because the source may or may not have contact. In reality, the receiver may be able to position and reposition himself for optimum tracking and weapons deployment. (2) A submarine that is alerted, feeling threatened by the active source, will typically maneuver to present a minimum aspect (bow or stern), to the active unit; a move which may provide the receiver with a more favorable aspect. (3) The tactical use of bistatics is also compatible with the current thrust of surface ASW toward passive engagement. Of course, against a diesel-electric submarine, active sonar must be employed and bistatics may provide optimum utilization of the active ping against such a quiet target. (4) A potential tactical advantage involves the utilization of a direct support submarine with a task force. With the submarine as the receiver and a surface or airborne unit providing the active illumination, the submarine may covertly obtain a reasonable picture of surface and subsurface contacts.

Disadvantages of these types of tactical employment are several: The utility of bistatics is limited by the accuracy of navigation between the source and receiver(s). Contemporary systems have capabilities superior to those of earlier bistatic testbeds for determining the relative positions of source and receiver. These data must still be processed and

available to both units as either or all of the participating units may not have this capability. This introduces the problem of communications to the bistatic operation. Both units will be constantly communicating information with respect to their position and the target's position which may create a burdensome and confusing communications situation. Bistatics, utilizing existing equipment, may also be limited by operator perception. The human operator must be able at a minimum to distinguish the bistatic echo from the transmitted signal and from any other sources of interference. In a worst case, he may also have to record the difference in time between the receipt of the source's transmission and the returning echo. This time interval is the basis for bistatic range determination.

Bistatics may also prove to be disadvantageous against coated or otherwise treated targets. If the net effect of the treatment is the reradiation of the incident sound energy (i.e., forward scattering instead of backscattering), the chance of having the passive receiver in an advantageous position is improved. Likewise, bistatics may allow faster and more accurate target classification. If the bistatic geometry and target orientation allow return echoes to be registered by both the source and the receiver, the net result is the potential doubling of the data rate of target information. Multiple receivers would result in multiple improvement in this data rate.

Bistatics could also prove to counter some of the advantages the submarine is capable of achieving with respect to physical oceanography. A conspicuous example of this utilization would be in the area between bottom bounce and the convergence zone detection regions. As depicted in Figure 2, sufficient energy still exists in this region following incidence of source acoustic energy on the target, to generate an echo. If the bistatic receiver were in this zone, it could take tactical advantage of the echo. Although the geometry of this application appears complex, actual employment of this tactic would be quite simple: The receiver would reposition until the signal from the source and the echo to the receiver were essentially coincident; it would then be directly over the target. This technique could also be employed in searching for targets that are utilizing an oceanic thermal front as an acoustic screen. (Figure 3) With the source on one side of the front and the target on the other side, it may be impossible for the source to detect the target. However, the source could ensonify the target's side of the front by utilizing bottom bounce or convergence zone modes. If the receiver were positioned on the target side of the front, it could utilize the sound energy to detect the target. Possible drawbacks to these applications include the difficulty in determining the precise location of an oceanic front and the requirement for the receiver to remain undetectable by the target submarine. If the receiver is detected, the

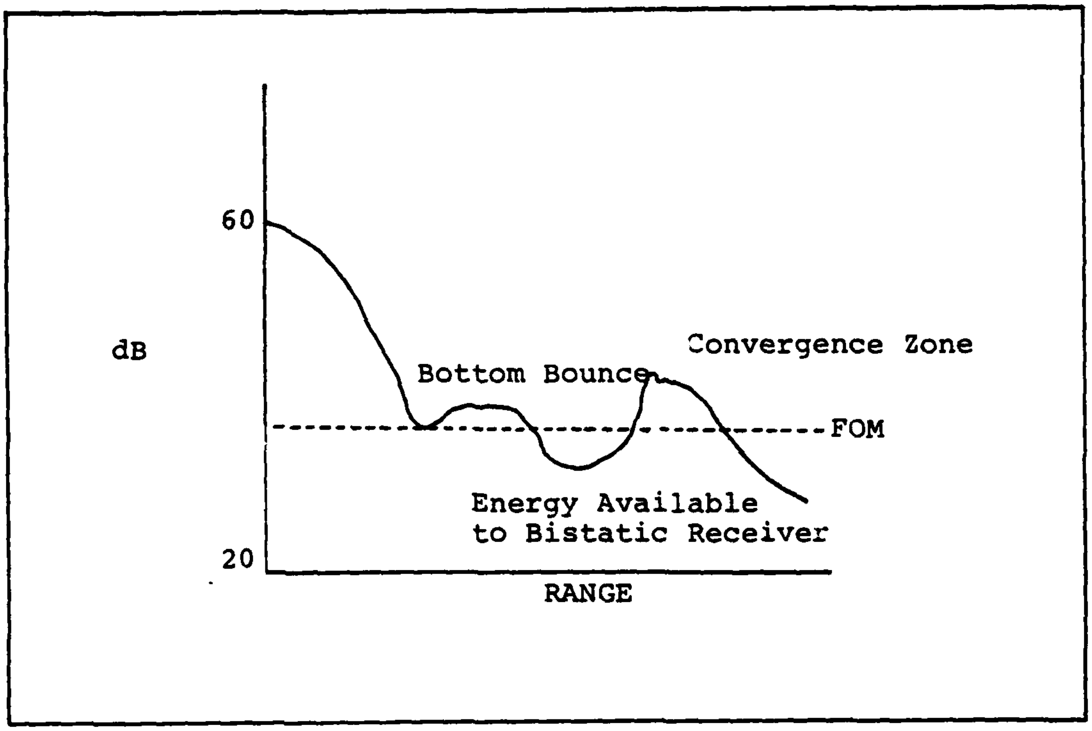


Figure 2. Bottom Bounce-Convergence Zone Detection

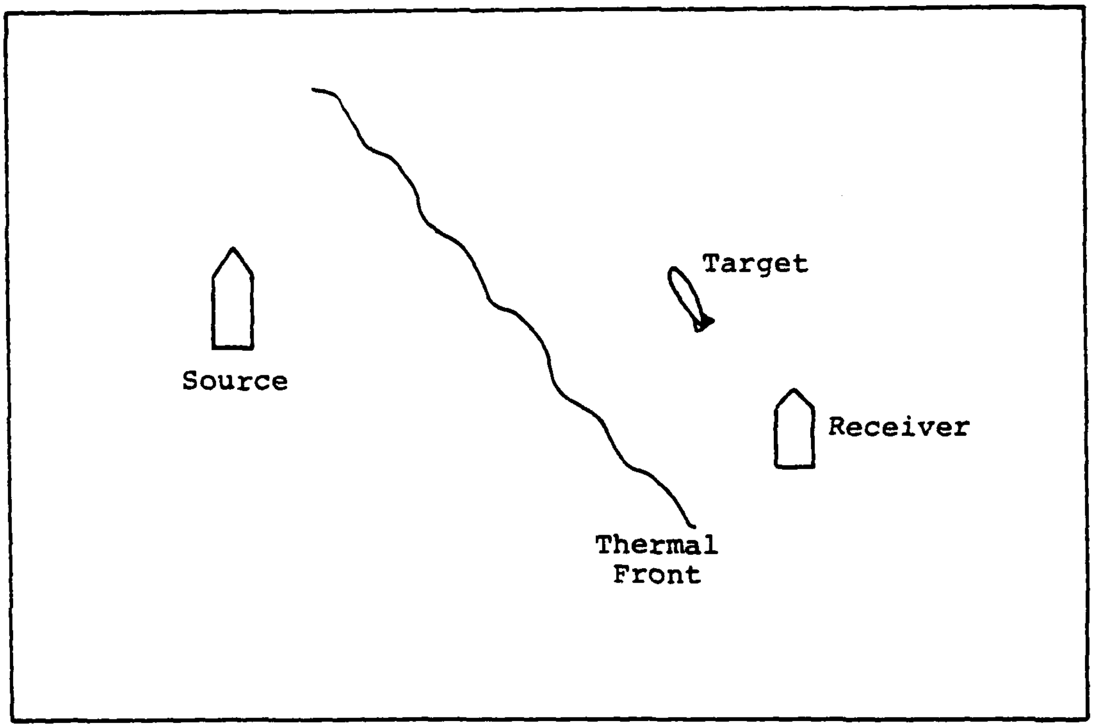


Figure 3. Thermal Front

submarine will be in a typical "1 vs 1" passive tracking encounter and will most likely take appropriate action. The use of rapid response, variable depth sonar vehicles in the receiver mode (i.e., SH-3 Helicopters) may negate this action.

The greatest advantages of bistatic employment concerns: (1) the ability to track the target submarine passively, (2) the element of confusion generated for the target in that it does not know in which direction its greatest threat lies, (3) possible improvement in classification, (4) denial of some forms of target acoustical treatment and countermeasures. These are all elements of tactical advantage or force multiplication.

Disadvantages of bistatic utilization stem primarily from the fact that it is not a use-on-demand technique. Sufficient pre-planning must exist with respect to tactics, navigation, and communication.

C. HISTORICAL BACKGROUND

The concept of utilizing a receiver separated from a source in a detection system did not originate in underwater acoustics. In fact, the contemporary concept of monostatic radar actually has its roots in bistatic radar experimentation. The early radar prototypes, called wave interference radars, consisted of separated transmitters and receivers. The first demonstration of bistatic radar as a means of detecting ships was done by the Naval Research Lab in 1922. By the late

1930's, monostatic radar had evolved from the bistatic systems and was found to be more effective in the applications important at the time. Bistatic radar was briefly revived in the 1950's and early 1960's when it was found that in particular geometries, the target cross-section was actually greater than in monostatic systems. The application of a bistatic radar system is mentioned occasionally in contemporary literature, particularly as a means of countering the "stealth" technology for making aircraft less detectable by monostatic radar.

Bistatic sonar was developed in the 1950's by civilian laboratories working for the Office of Naval Research. Most work was centered around the concept of expanded detection areas using shipborne and fixed sonars. Later experiments involved ship-to-sonobuoy applications and the development of bistatic specific arrays. Like bistatic radar, bistatic sonar was not considered as versatile or as effective as its monostatic counterpart and interest fell off in the late 1960's. Little research was done in this field in the ensuing years though it now appears that the Soviets may have been actually using bistatic systems in antisubmarine efforts for several years.

Recently, western researchers, particularly those associated with NATO, have expressed renewed interest in bistatic applications. Motives are diverse but are no longer centered around increasing the area of coverage, and the platforms

utilized include conventional surface ships, surface effect ships, submarines, and aircraft.

II. PERTINENT SONAR EQUATION PARAMETERS

A. GENERAL SONAR EQUATION

In consideration of the physical elements involved in the generation and utilization of underwater sound and of the effects of the medium upon this sound, a series of basic equations was evolved in an attempt to relate the various parameters in a manner that would be useful in sonar design and prediction. These parameters of sound and sound generation are combined to form what is called the sonar equations. The equations were originally useful as a means of evaluating the performance of a particular sonar with respect to its maximum range of detection and were later developed into a tool for the sonar designer. For the purpose of analyzing bistatic echo-ranging, the equations will be utilized in their original concept, as a means of determining the performance of a particular sonar system; in this case the transmitter and receiver of a bistatic sonar arrangement. Because of the absence of a standardized notation, we shall use that found in Urick's Principles of Underwater Sound.

Much of the development of underwater acoustics is based on equating acoustic parameters with their atmospheric electromagnetic counterparts, particularly the parameters of radar. The sonar equations are not exceptions. The underlying concept behind the sonar equations is the equality at a certain point between the desired portions of the acoustic

signal and the undesired portions of that same signal at the receiver at the time of detection. The desired portion of the acoustic signal is that part that can provide knowledge of the existence of the target and is called the signal. The undesired portion includes naturally occurring sound and sound resulting from the interaction of the acoustic pulse with the medium and is called background. This background tends to mask or obscure the presence of the signal from the observer until the level of the background and signal are equal. From this point, the signal tends to override the effects of the background and target detection is possible. A sonar therefore, is at the lower limit of achieving its design goal when the signal level is equal to the background masking level. NOTE: The term masking applies only to that part of the background that is in the frequency bandwidth of the receiver at the time of reception. Masking further applies to the way the received signal is processed and the measures used for probabilistically determining that the received signal represents a detection or no detection.

Background can be broken into two components: (1) noise which is basically steady-state and either ambiently generated or created by the receiving platform and, (2) reverberation, a function of active sonar, which is not steady-state but exhibits some rate of decay and is a product of the interaction of the acoustic pulse with the various scattering elements inherent to the medium. With respect to the signal

and the background, Urick has broken the sonar parameters into three categories corresponding to those determined by the equipment, the medium, and the target (Table 1). When used in the sonar equation, these terms represent levels and therefore can be expressed logrithmically so that the terms can be combined by simple addition. The units of the parameters are decibels. The sonar equation exists with respect to either passive or active sonar application.

In terms of signal and background, the basic sonar equation is

$$EL \geq ML \quad \text{[EL is echo level, ML is masking level]},$$

where a 50% probability of detection for some stated probability of a false alarm just occurs when the levels are equal. The elements that comprise the masking level, ML, as previously expressed, depend on the nature of the noise involved and the type of sonar employed. Primarily, masking level can be decomposed into the sum of the terms detected noise level, DNL, and detection threshold, DT. DNL is the level of the undesired portion of the received signal and DT is the means of introducing an element of confidence or probability into the correlation between received signal and target presence. If noise is the predominant background,

$$DNL = NL - AG$$

where NL is the noise level and AG, array gain which involves the directionality of the sound and the receiver and the capabilities of the signal processor. If reverberation

TABLE 1

SONAR EQUATION PARAMETERS

EQUIPMENT PARAMETERS:

Projector Source Level	:	SL
Self-Noise Level	:	NL
Receiving Directivity Index	:	DI
Detection Threshold	:	DT

MEDIUM PARAMETERS:

Transmission Loss	:	TL
Reverberation Level	:	RL
Ambient Noise Level	:	NL

TARGET PARAMETERS:

Target Strength	:	TS
Target Source Level	:	SL

From: Principles of Underwater Sound by Robert J. Urick

predominates,

$$DNL = RL$$

where RL is the reverberation level.

With respect to the signal, and therefore the echo level, EL, a primary factor is the level of the acoustic pulse, whether generated by the active transmitter or by the target itself. This is known as source level, SL. The losses experienced by this pulse propagating through the medium form another factor of echo level called transmission loss, TL. Transmission loss occurs in each direction of active propagation. A third factor, useful in active applications, is target strengh, TS. This term expresses the level of sound energy reflected or reradiated in the direction of the receiver compared to that incident upon the target. These terms can be combined so that:

$EL = SL - 2TL + TS$ for active consideration and

$EL = SL - TL$ for passive sonar applications.

Relating signal to background in terms of the basic sonar equation (EL - ML), the expression for monostatic active sonar becomes:

$SL - 2TL + TS \geq NL - AG + DT$ for noise limited conditions,

$SL - 2\ TL + TS \geq RL + DT$ for reverberation limited conditions, and

$SL - TL \geq DNL + DT$ for passive sonar.

B. ANALYSIS OF TERMS MOST AFFECTED BY BISTATIC OPERATIONS

In applying the sonar equations to the bistatic case, manipulation of the equation for passive sonar combined with the fact that the target acts as a reradiator of impinging acoustic energy from the source results in an expression for bistatic echo-ranging,

$$EL = SL_{\text{transmitter}} - TL_{\text{transmitter-target}}$$
$$- TL_{\text{target-receiver}} + TS$$

It is apparent that the major deviation from the signal side of the monostatic equation involves the potentially different values for transmission loss. These two values, TL and TL', will depend on the different ranges between the source-target and the target-receiver and the elements of transmission loss that may affect sound propagation over these ranges. The generation of these two transmission loss values is of insignificant difference from the monostatic values in degree of difficulty.

Another variation between the monostatic and bistatic case involves the level of target strength. Existing tables of values for target strength, with respect to the several factors involved, may be inaccurate for bistatic geometries. The last term on the echo level side of the sonar equation is not sensitive to monostatic or bistatic configurations. Source level depends on the particular active sonar used as the illuminator whether it is designed to also receive the returning echoes or not.

On the masking level side of the sonar equation, both array gain and detection threshold are obviously independent of the geometry of the source and receiver positions. Noise level, whether primarily ambient or self noise based, is still a function of the physical construction of the receiver and not the geometry of the system. However, noise level could become a complex problem if the bistatic ranges were so great that different states of turbulence, shipping, or wind were encountered between the source and receiver. For surface duct, the noise level should vary insignificantly between monostatic and bistatic cases.

The term most affected by the bistatic geometry would be the reverberation level. In the monostatic case, the major lobe of the source is colinear with the major lobe of the receiver (since they are essentially the same), and the relevant reverberation volume may be fairly easily calculated. The reverberation volume is then used to calculate the reverberation level by accounting for the scatterers most likely contained in the volume. Bistatic reverberation volume is more complex to calculate than the monostatic volume because of the nature of the geometric variations existent in the bistatic case. Therefore, the terms of the sonar equation most affected by bistatic operations are target strength and reverberation volume.

C. SIGNIFICANCE OF BISTATIC REVERBERATION LEVEL AND TARGET STRENGTH

When an acoustic wave passes over a particle in the water, the particle is caused to vibrate by the incident energy and will, as a result, become a secondary source of sound. Accordingly, the intensity of the sound generated by the second source is proportional to the intensity of the incident sound. The reradiated sound is called reverberation.

Reverberation occurs when there is a sufficiently strong sound field of relatively low directivity generated by the primary source and a sufficient amount of scatterers in the vicinity of the target. These scatterers may be air bubbles, fish, plankton, solid particulates, or physical inhomogeneities. The sea surface and sea bottom may also serve as sound scatterers. Reverberation is, therefore, the sum of scattered sound or echoes that may be in competition at the receiver with the desired target echo. This scattering begins with the incidence of sound on a particular scatterer and ends when the sound energy is no longer incident. It is presumed that multiple scattering has a negligible effect on the overall reverberation level because of the small intensities involved in the rescattering of scattered sound. Reverberation is a phenomenon of active echo-ranging. It is dependent upon the pulse length of the signal, on the directivity of both the source and the receiver, and on the geometry of the situation. There are primarily two types of reverberation: volume, which

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deals with three-dimensional regions of scatterers in the ocean and surface, which generally includes the scattering effects of the sea surface or the sea bottom.

There are two important factors in the calculation of the reverberation level encountered in a particular situation. The first is the scattering strength, S_v for volume reverberation and S_s for surface reverberation. This is another term with origins in the radar equation and it involves the backscattering cross-section per unit volume or area. Expressed as a level, scattering strength is the ratio of the intensity of sound scattered by a unit volume or area to the intensity of the sound incident upon the scatterers. Representative values for scattering strength exist for different water masses and propagation conditions.

The second important factor in calculating the reverberation level is the volume or area over which the reverberation in competition with the target echo takes place. Essentially, this problem can be reduced to geometric terms in relating the major lobe of the source with that of the receiver. In the monostatic case, it is assumed that the major lobe of the source completely overlaps that of the receiver. In this case, the width of the reverberating region about the target is $^{c\tau}/2$, where c is the speed of sound in the medium and τ is the pulse length of the echo-ranging signal. The effect of the range of the target and reverberating region is accounted for as transmission loss via

geometrical spreading, expressed as TL_g. The solid angle subtended by the major lobe is another element of calculation. For volume reverberation in the monostatic case

$$10 \log \text{Volume} = 10 \log {}^{c\tau}/2 + TLg + 10 \log \Omega$$

which leads to an expression for the target strength of the scatterers, TS',

$$TS' = S_v + TL_g + 10 \log \frac{4\pi c\tau}{2} - DI.$$

For surface reverberation, the area is a function of the horizontal angular width of the source radiation pattern, the range to the area, and the pulse length,

$$A = \frac{1}{2} \; r\Theta c\tau,$$

which results in a scattering target strength,

$$TS' = S_s + 10 \log r + 10 \log {}^{\Theta c\tau}/2.$$

Simply expressed, $TS' = 10 \log V + S_v$, for reverberation and,

$TS' = 10 \log A + S_s$, for surface reverberation.

In the calculation of reverberation level, the primary difference between monostatics and bistatics is the manner of determining the reverberant volume or area. This calculation becomes an exercise in solid geometry: ascertaining the volume that surrounds the target returning reverberation in competition with the echo and also within the intersection of the beam representing the major lobe of the source with the beam representing the major lobe of the receiver. The geometry is particularly sensitive to the respective ranges of the source and receiver and the angle separating the beam

from source to target from the beam between target and receiver. Masking by reverberation becomes more significant as this angle increases, out to a limit.

Target strength refers to a characteristic of the specific target sought in the echo-ranging process. It is essentially a measure of the reradiating or backscattering capability of the target. The target strength of the target is measured with respect to the reradiated sound in the direction of the receiver by utilizing the ratio of the intensity of the reradiated sound, extrapolated back to a distance of one meter from the target, to the intensity of the sound incident to the target,

$$TS = 10 \log \left. \frac{I(r)}{Ii} \right|_{r= 1m}.$$

Target strength is a function of the submarine class, speed, range, aspect, and the pulse length and possibly frequency of the echo-ranging sound. For simple structures such as spheres and flat plates, target strength may be easily calculated. However, the complex construction of a submarine makes the computational value of target strength difficult to obtain and suspect in precision. Instead, IN SITU measurements of target strength are usually made on actual submarines. The results are highly variable but do present approximate limits and representative values of target strength for a particular submarine.

Very little is known about the effect of bistatic geometry upon the perceived target strength of a particular target.

Only recently have actual measurements been attempted on submarines and scale models of submarines. In the monostatic case, the intensity calculations for incident and reradiated sound are based upon measurements from colinear points representing the same path from source to target as from target to receiver. Bistatics, by definition, involve different transmission paths which may include variations in the actual reradiating process to create an echo in a direction other than that of the incident energy. There may be additional effects based on the angular separation of the source and receiver beams. Existing data indicate that bistatic geometry does lead to different values of target strength than those observed monostatically. There may exist techniques that enable bistatic target strength determination based on principles of radar or physical optics which utilizes a manipulation of monostatic values.

III. BISTATIC REVERBERATION LEVEL

A. MODEL DEVELOPMENT AND ASSUMPTIONS

In the calculation of the reverberation level that may be expected to be encountered in a bistatic echo-ranging problem, the most significant variable to be determined is the volume of water containing scatterers that can compete with the target echo. An analysis of this volume is straightforward in the monostatic case but is considerably more difficult in the bistatic geometry.

When an active source transmits acoustic energy in the form of a pulse into the water, the range from the source to the pulse front at a given time is a factor of the speed of sound in the particular water mass and the elapsed time since transmission. With an omnidirectional source, the acoustic pulse forms a sphere about the source that expands symmetrically with time. In the monostatic case, if a target is encountered by the pulse, the range to the target from the source can be expressed as $r_t = {}^{ct}/2$, where c is the local sound speed and t is the time from signal transmission to echo detection. The only scatterers that will compete with the echo from this target at the receiver are those in the vicinity of this range. To further define the extent of the effective reverberation, it is necessary to include the duration of the acoustic pulse in the range calculations. The echo generated by the target will, when it reaches the receiver, last for a time dependent

on the pulse length. Therefore, any scatterers which contribute to reverberation arriving at the receiver during the receipt of the target echo are important. The region in which these contributing reverberators exist is the thickness of the reverberation volume. Given the sound speed, c, and the duration of the acoustic pulse, , this thickness can be determined to have a value of $c\tau/2$ [Ref. C]. Therefore, in the monostatic case, all scatterers producing competitive reverberation can be considered to lie between the surfaces of two expanding spheres: An inner sphere of radius

$$r_i = c(t/2 - \tau/4),$$

and an outer sphere of radius

$$r_o = c(t/2 + \tau/4).$$

A target echo encountering reverberation would lie in the middle of the volume between the two spheres. Specifically, in the case of omnidirectional, geometric, ideal propagation, the reverberation volume can be depicted by three concentric spheres of radii r_i, r_t, r_o proceeding outward from the source and expanding with time. Monostatic volume can be further limited if the source or receiver are directional.

Of the various techniques for calculating bistatic reverberation volume, the most promising solution may lie in the application of the geometric properties and characteristics of the ellipse to the bistatic engagement. As an overview, this process develops several ellipses from specific characteristics of the bistatic geometry and utilizes the properties

of these generated ellipses to calculate the desired volume. The spatial aspect considered initially is the horizontal section, or planview, in which the source, target, and receiver are depicted in the same plane. In the final calculation of the reverberation volume, the dimension derived from this aspect will be the thickness of the area. The cross-sectional area, which provides the other two dimensions of the volume, is perpendicular and symmetric to this planview and is determined at the point of the target. Properties of the ellipse which will be utilized in this development are discussed in Appendix A. The development of this concept requires the use of several assumptions and borrows occasionally from the previous treatment of the monostatic volume. If, in the most general case, the bistatic sonar is considered as a means of utilizing the maximum range of sound transmission from a particular source with given environmental conditions, this optimal range is considered analogous to the major axis in ellipsoidal geometry and thereby serves as the basis for subsequent development. The locus of possible points representing a target's position for a specific bistatic range traces out a prolate ellipsoid in three dimensions (Figure 4). The limiting case of this solid occurs when the source and receiver are coincident (monostatic), in which case the sphere discussed previously results. The two-dimensional figure resulting from the planview section of this solid is an ellipse.

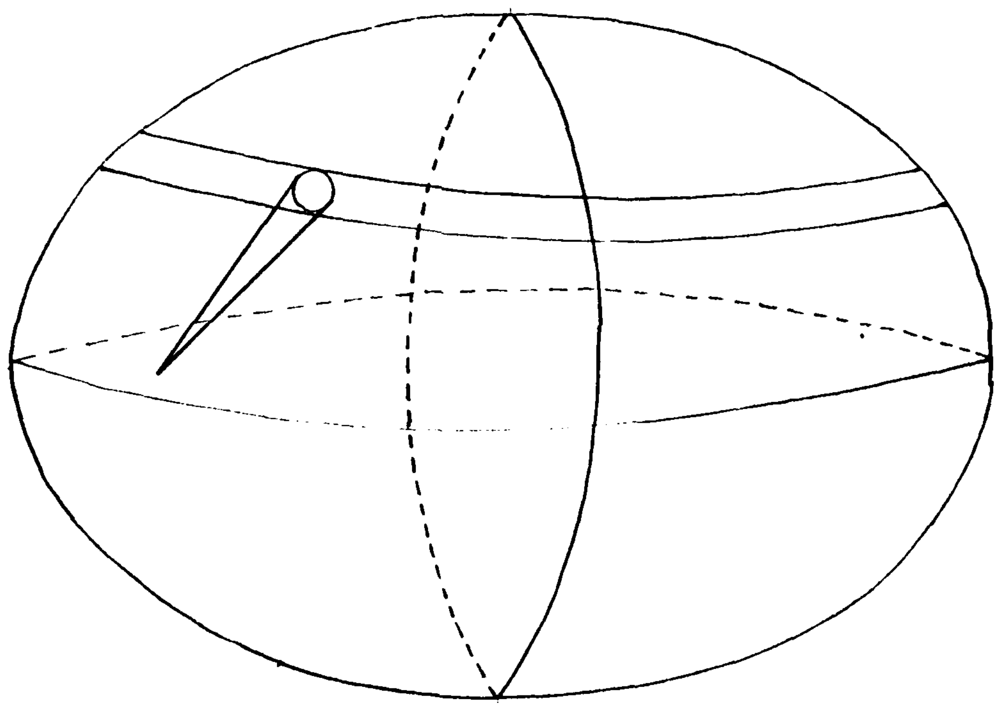

Figure 4. Prolate Ellipsoid

Several assumptions allow for the generation of an appropriate solution for the volume. This general solution lends itself quite handily to modification when the assumptions used are not practical, too simple, or too restrictive for specific situational calculations. The assumptions include: (1) the source transmits a pulse with some fixed vertical dimension. This results in an ensonified band around the ellipsoid containing the target. After the general solution is proposed, this assumption may be dropped and a directional source of specific beamwidth utilized in the calculation, (2) the sound spreads geometrically without horizontal refraction from the target to the receiver. When directional sources are considered, this assumption is true also for spreading from the source to the target. This implies cone-shaped beams of sound with vertices at the directional receiver and/or source. These cones have a beamwidth predicated on the half-power points of the major lobe of propagation. This beamwidth is equal to the generating or vertex angle of the propagation cone, (3) the effective horizontal beamwidth is small ($\leq 20^{\circ}$).

The ellipse containing the source, receiver, and target will be called the range ellipse. The position of the source and receiver represent the foci of the range ellipse. Further assumptions based on this particular geometry include: (4) the target is a point on the perimeter of the range ellipse and has no dimensions thereby eliminating any consideration of time stretching, (5) the bearing of the source to the receiver

is known, (6) the range from the source to the receiver is known, (7) the range from the source to the target or the receiver to the target is known.

The basic features of the ellipse can now be correlated with the geometry of bistatic reverberation volume. In the basic bistatic case (considered basic because it involves one transmitter, one target, and one receiver, as in Figure 1), the position of the source and receiver, as stated previously, correspond to the foci of the ellipse. The locus of possible target positions which would yield return echoes either to the receiver or to the transmitter and receiver together, forms the perimeter of the range ellipse. The total range, R is equal to the sum of range R_1 and R_2 where R_1 is the source-to-target range and R_2 is the target-to-receiver range. This range is equal to the length of the major axis of the range ellipse, i.e., $R = 2a$, and R_1 and R_2 are the focal radii. The major axis is a function of target position to the limit of the maximum range sound will travel under the given environmental conditions. When the generated sound energy is insufficient to reach the target and reradiate to the receiver, the range ellipse will not exist. At the other extreme, the ellipse will not exist when the target is colinear and between the source and receiver. Since the target is assumed to be a point on the range ellipse, and the tangent to the ellipse at that point approximates a segment of the ellipse, this tangent line will intersect the directional

cone(s) of sound propagation at the target. The intersecting tangent line is contained in a plane perpendicular to the plan-view geometry. The intersection of this tangent plane with the cone of sound transmission forms a conic section. A plane section which makes a slightly smaller angle than perpendicular with the cone's axis is an ellipse. Specifically, an ellipse is formed if the plane which establishes the conic section makes an angle with the conic axis which is greater than the conic vertex angle. If source and receiver are both directional, the conic section resulting in an ellipse is formed by the intersection of the plane tangent to the range ellipse at the target point and the cone of sonic transmission that has the smallest dimensions measured at the target point.

To summarize the developments to this point, at appropriate ranges between source and receiver and between target and source/receiver, the range ellipsoidal surface can be approximated by its tangent plane at a point of the ellipse. Intersection of this plane with the cone of sound ensonifying the target will generate a conic section. This conic section will, for most cases, be an ellipse. This conic section will be referred to as the target ellipse.

The cone of sound transmission and the intersecting ellipse-plane segment is depicted in Figure 5. By geometric definition, the point location of the source or the receiver is the conic vertex, here designated V. The line from the vertex about which the cone may be generated by revolution, and

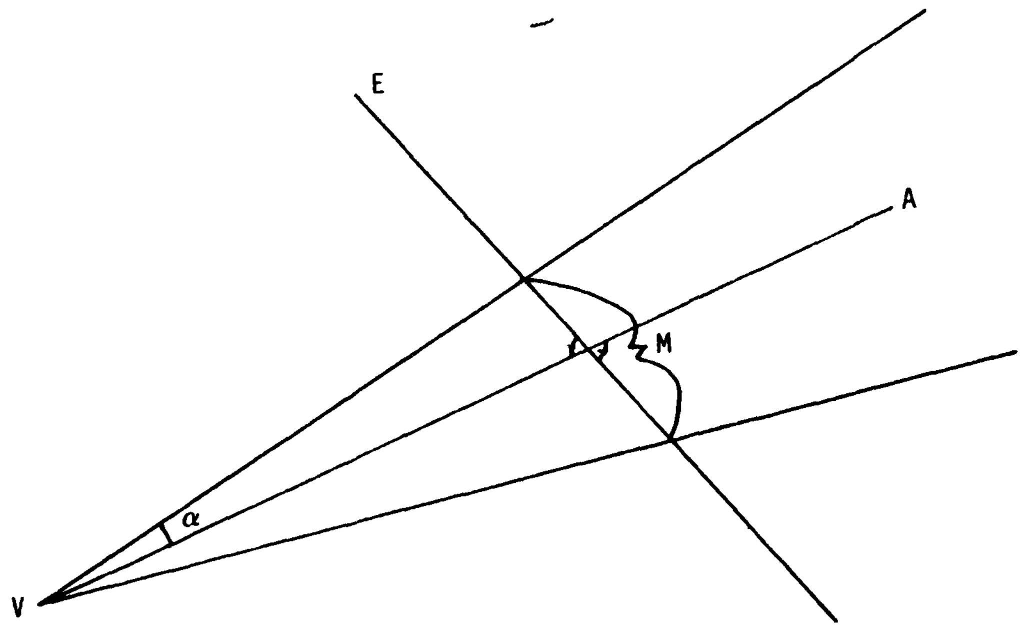

Figure 5. Conic Section

therefore about which the cone is symmetric, is the axis, A. The constant angle between the axis and the surface of the cone is the generating or vertex angle, α. For the purposes of this development, the length of the axis represents the range of the target from the source or receiver. The vertex angle is one-half of the effective beamwidth of the main lobe of the generation pattern of either the source or the receiver.

Figure 5 also depicts the intersection of the segment of the ellipse at the target, approximated by the plane containing the tangent to the ellipse at this point. The plane, designated E, forms a conic section with the cone. This section is an ellipse when the angle between the axis, A, and the plane, E, is greater than the vertex angle. If this angle is a right angle, the special case of a circle, will be generated. The length of the planar segment intersecting the cone is the length of the major axis of the conic section previously designated the target ellipse.

The major axis of the target ellipse, designated M, is the tangent approximation of the arc-length of the range ellipse intersecting with the cone of transmission. The minor axis of the target ellipse, N, will be a function of the radius of the circular cross-section of the intersecting cone of transmission. If both source and receiver are directional, N will be developed from the radius of the cone having the smaller dimensions at the target point (Figure 6). The area of the target ellipse is the cross-sectional area of the

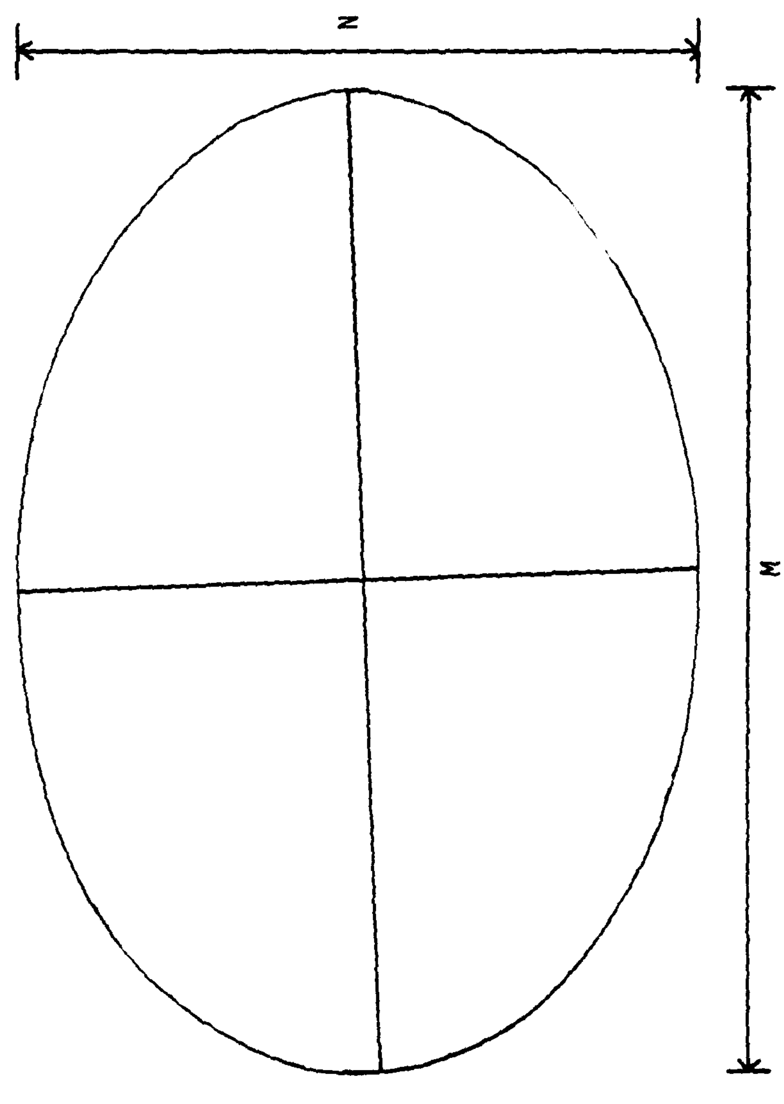

Figure 6. Target Ellipse

reverberating volume measured at the mid-point or target position. This area will be integrated over the third-dimensional component, the thickness, to determine the volume. As presented in Appendix A, the area of the target ellipse is,

$$A = \frac{\pi MN}{4}.$$

The mathematical determination of the semi-major and semi-minor axes is developed in Appendix B.

Returning to the assumptions inherent in the concept of the range ellipse, the same technique may be used to postulate the reverberation volume of the omnidirectional source at any given time (and therefore range), of acoustic pulse transmission. In a description similar to that of the monostatic source, the *range* ellipse would lie midway between two ellipsoidal surfaces representing the volume of reverberation in competition with the target echo. As in the monostatic case, the thickness of this area will be a function of the pulse duration and the sound speed. An equivalent approach is to consider two ellipses, one within the other, with the distance of separation between the ellipses derived from $\frac{c\tau}{2}$. The semi-major axis of the inner ellipse, which is the ellipse containing the target is

$$r_I = ct.$$

The semi-major axis of the outer ellipse, which combines the effects of the total region of reverberators, is

$$r_o = c(t+\frac{\tau}{2}).$$

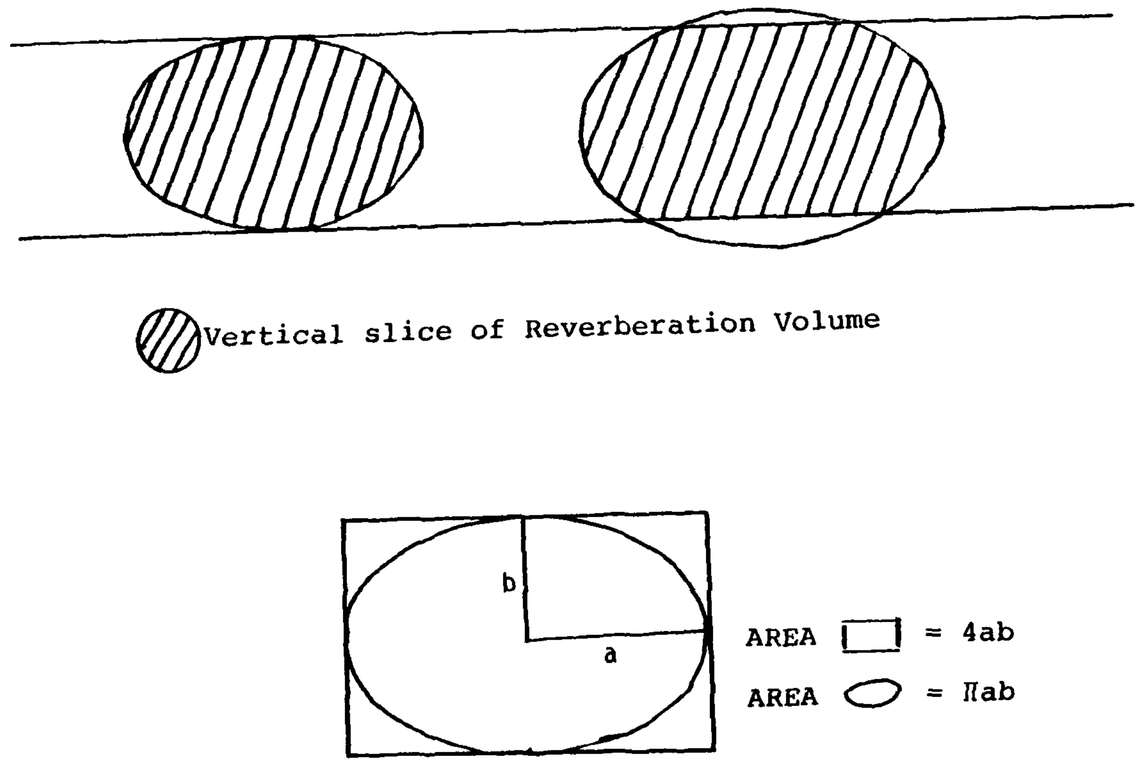

Figure 7. Target Ellipse as a Limit

The term t represents the total elapsed time from transmission by the source until reception of the reflected echo by the receiver.

The limiting case of the thickness of the volume is $^{c\tau}/2$, and occurs as the bistatic geometry approaches monostatic conditions. For other cases, the thickness of the volume is a function of $\frac{c\tau}{2}$ $f(c\tau/2)$. A close approximation to the actual reverberating volume can be calculated from the expression,

$$V = (\Pi)\ (\tfrac{M}{2})(\tfrac{N}{2})\, f(^{c\tau}/2).$$

The development of the function of $^{c\tau}/2$, $f(^{c\tau}/2)$, with respect to volume thickness, is found in Appendix C.

With an omni-directional source sonically illuminating a strip of the ellipsoid and the cone of transmission of the directional receiver intersecting this band at the target, further complications may arise. If the target ellipse lies completely within the source band at the target, implying that the minor axis of the target ellipse is the diameter of the receiver's cone at the target, then the cross-sectional area of the reverberant volume is equal to the area of the target ellipse as previously postulated.

If the target ellipse is greater than the source band, an ellipse is still a good approximation of the volume's cross-sectional area. In this case, the minor axis of the target ellipse is the diameter of the source beamwidth. As indicated in Figure 7, the extreme case of this type intersection would

be a rectangular figure with the elliptical-type ends. The area of a complete rectangle of these proportions would be 4ab. The area of an ellipse within this rectangle is Πab. The actual area lies between these two values but the difference in limits accounts for less than one decibel of reverberation level.

If the receiver and source are both directional, then the minor axis, N, is simply a function of the diameter of the smaller of the cones at the point of intersection.

B. CALCULATION OF REVERBERATION LEVEL

The target strength of the competing volume of scatterers, TS', is calculable given the solution for the bistatic reverberation volume. Since the scattering target strength is equal to

$$TS' = 10 \log V + S_v$$

and representative values of scattering strength, S_v, are easily obtained, the logarithmic value of the volume will complete the solution. The scattering target strength is

$$TS' = S_v + 10 \log \frac{\Pi MN}{4} f(c\tau/2).$$

The elements of range and directivity that are evident in the monostatic expression of TS' are contained in N. The resulting reverberation level is

$$RL = SL-TL-TL'+S_v+10 \log \frac{\Pi MN}{4} f(c\tau/2)$$

C. SURFACE REVERBERATION

An approach similar to that for determining volume reverberation can be used to develop a value for surface reverberation. Surface reverberation can be treated as a special case of volume reverberation in that the vertical depth of the volume is insignificant with respect to the area of scattering competitive with the target. The assumptions inherent in the development of the volume are also applicable to the area, particularly the assumption that the beams from source and receiver are straight because of the ranges involved. With the additional assumption that the source and receiver are at relatively shallow depths, then the intersection of the beams with the surface will be at small angles.

The limits of the reverberating area are with respect to the bistatic geometry and the intersection of the sonic beams with the surface. If the major axis of the target ellipse is integrated over the volume's dimension of depth, the resulting two-dimensional area will be the maximum value of reverberant area. This area is used in the solution when the geometry of the beam-surface intersection is such that the entire bistatic area lies within the region of intersection. When the area common to the beam and the scattering surface is smaller than the bistatic area, then the overlap between the two regions is used in calculations. Manipulation of the two generated areas to establish a value

for use in reverberation level calculations is not difficult. However, since the region of surface-beam intersection is situationally specific, a more specific expression for the target strength of the scatterers is not developed.

IV. BISTATIC TARGET STRENGTH

The effect of the bistatic geometry on the perceived target strength of a submarine remains one of the least developed aspects of bistatic sonar application. It may have possibly been concluded by bistatic sonar pioneers that the variations in target strength that might result from using a separated source and receiver are insignificant when incorporated with the margin of error inherent in determining monostatic values. The best technique for measuring target strength involves actual IN SITU measurements using a given target. The very nature of acoustic propagation in the water is variable enough to complicate these measurements. In addition, the target strength may be dependent on the individual target to the point that measurements of other submarines, even those of the same class, vary by a few decibels. Other factors contributing to the variability of target strength measurements include aspect, fluctuations, altitude angle, target range and speed, ping length, and frequency of the source. All of these factors contribute to the margin of error in measurement of which bistatic geometry may be just another element. Measurements made using optics, as was originally done, and acoustic models, which is the contemporary approach, add the uncertainty of the effect of scaling to the derived measurements.

Regardless, in keeping with sonar tradition, application of bistatic radar techniques in generating target cross-sections is likely to correspond to methods for producing bistatic target strength values of reasonable accuracy. Crispin, Goodrich and Siegel [Ref. 3] proposed a theoretical means of determining bistatic radar cross-sections for targets with sufficiently smooth bodies. Essentially, the bistatic cross-section for a particular set of transmitter and receiver directions is equal to the monostatic cross-section for the vector sum of these directions. This is applicable when the transmitter vector is not equal to the receiver vector and the angle between these vectors is not 180°. This concept has been refined to the bistatic theorem mentioned by Urick [Ref. 2] which states that the bistatic target strength is equal to the monostatic target strength measured at the bisector of the bistatic angle, the angle between the source-to-target ray and the target-to-receiver ray. This theorem is applicable for bistatic angles of less than 180° and it actually approaches its limit of effectiveness in the vicinity of bistatic angles of about 150°. Urick adds the warning that the applicability of this theorem is questionable in the absence of measured data. In radar applications, the range of values derived bistatically are comparable with the values developed monostatically. Individual variations do exist, but on the average, monostatic and bistatic values of cross-section are roughly equivalent. This same conclusion may be true for bistatic sonar measurements.

To demonstrate the utilization of this approach in determining bistatic target strength, the classic "butterfly" monostatic target strength curves [Ref. 3] were converted into a simple computer program. For a given source aspect and receiver aspect, and therefore bistatic angle, a bistatic target strength level was generated. The results were used to create the graph of Figure 8. The graph is entered from either the top, for incident aspect, or left side, for receiver aspect. The line representing the aspect value is followed until it intersects with the other aspect value line. The cross-curve lines are then paralleled to the bottom or right-side of the graph where the value of bistatic target strength may be read. Simple interpolation is used for those values that do not specifically appear on the graph.

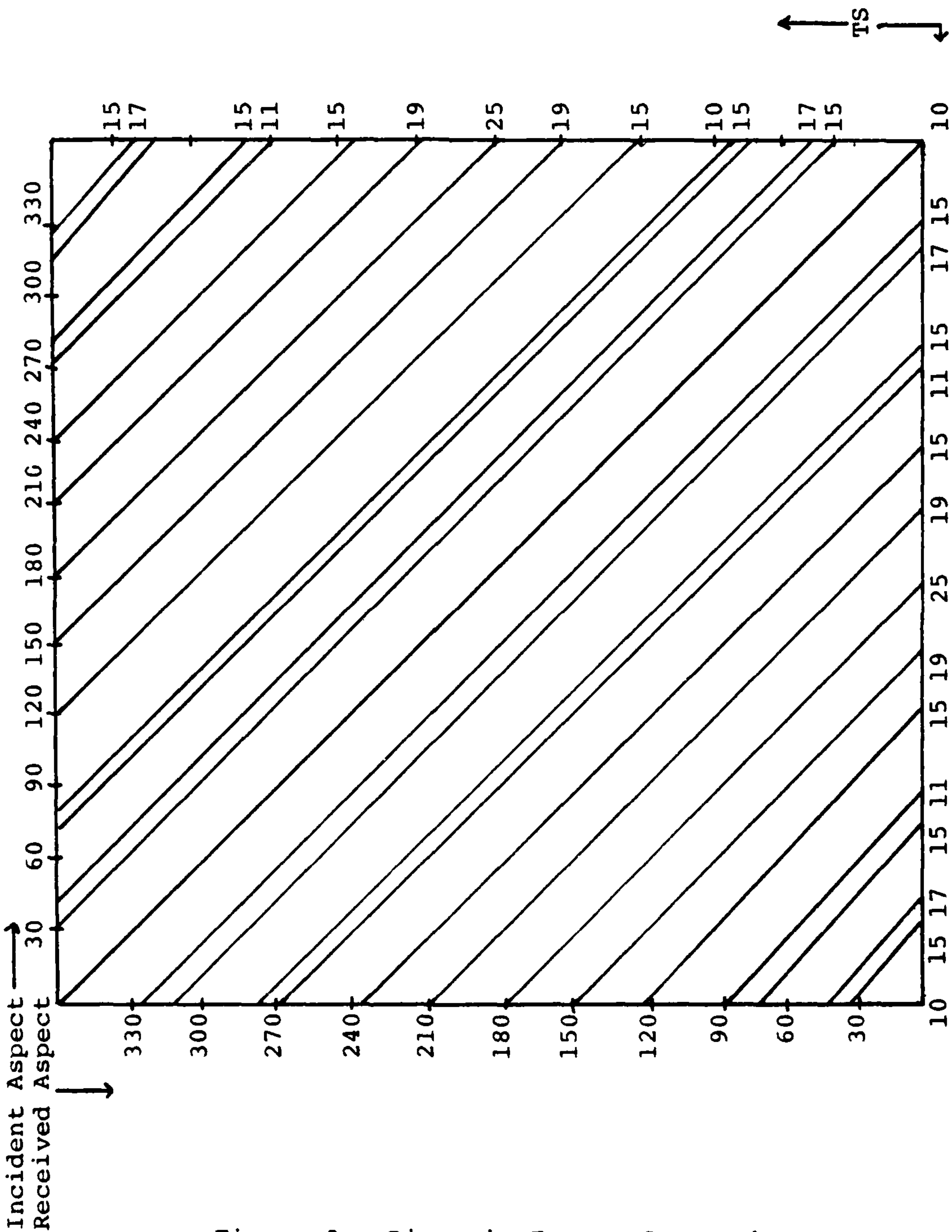

Figure 8. Bistatic Target Strength

V. CONCLUSIONS

The monostatic reverberation volume is expressed in terms of the cross-sectional area and the thickness of the volume of backscatterers. The expression developed in Chapter III for the bistatic reverberation volume can be reduced trigonometrically to

$$\text{VOLUME} = [\pi \mathfrak{z}^2]\ [\ell \cos(^C/2)]\ \left[\frac{\sec(^C/2)}{(1-\tan^2\alpha \tan^2(^C/2))^{3/2}}\right]$$

This expression is broken into terms similar to the monostatic elements,

VOLUME=[MONOSTATIC CROSS-SECTIONAL AREA][THICKNESS]
[BISTATIC ADJUSTMENT]

These three terms allow for direct comparison of monostatic and bistatic volumes.

The first term is the monostatic cross-sectional area of the smaller of the intersecting beams. In the case developed here, $\mathfrak{z}$, the radius of a right-circular cone, can be expressed with respect to the range and the effective horizontal beamwidth,

$$\mathfrak{z} = a \tan\alpha$$

The second term describes the effect of bistatics on the reverberation thickness. The thickness is a function of $^{c\tau}/2$ which is implicit in the variable ℓ, [Appendix C].

The bistatic adjustment is actually applied to the cross-sectional area to account for the modification of this area

resulting from the bistatic geometry. The adjustment is a function of the bistatic angle and the effective horizontal beamwidth of the beam of the smallest dimensions. The adjustment can, itself, be broken into two components; that of the span-wise dimension which is the length of the tangent segment within the sonic beam, and that of the altitude dimension which is a function of the width of the sonic beam used in calculations. The span-wise dimension will contribute an adjustment of

$$\left[\frac{\sec({}^{C}/2)}{(1-\tan^2\alpha\tan^2({}^{C}/2))}\right]$$

Regardless of the shape of the sound beam. In this development where conic beams were utilized, the "stretching" of the circular cross-section along the intersecting plane results in a ellipse of semi-major axis

$$\frac{M}{2} = \mathcal{Z}\left[\frac{1}{\cos(\frac{C}{2})\,[1-\tan^2\alpha\tan^2({}^{C}/2)]}\right]$$

The term $\mathcal{Z}$ represents the radius of the circular section of the cone at the target point. This term contributes to the cross-sectional area.

The width dimension may or may not contribute to the bistatic adjustment depending on the shape of the sound beam. Again, in the conic beam development, this dimension is the semi-minor axis of the target ellipse, ${}^{N}/2$. As discussed in Appendix B, the semi-minor axis is difficult to calculate but is a function of the radius of the circular cross-section

at the target point. This radius, δ, combined with the same term pulled from the semi-major axis, forms the cross-sectional area term ($\Pi\delta^2$ in the conic development). The contribution to the bistatic adjustment of the semi-minor axis becomes

$$\left[\frac{1}{[1-\tan^2\alpha\tan^2(C/2)]^{\frac{1}{2}}}\right]$$

For other sonic beams, i.e., a rectangular beam of sound rays, there may be a different value of bistatic adjustment. The rectangular beam would generate a section, with the interception of the beam by the plane, that would be a rhomboid.

The difference in the volume contribution to reverberation level from the monostatic equivalent can be derived from the thickness and adjustment terms. The sum of the ΔRL of these two terms is the total variation in the reverberation level as a result of the bistatic geometry.

In order to evaluate quantitatively the effect of the geometry on RL, a series of ellipses representing bistatic encounters were arbitrarily developed by using a common source-receiver range and various eccentricities. For each eccentricity, an inner and outer ellipse were constructed with a semi-major axis difference of $^{c\tau}/2$. The total range was held constant while the angle between the range vector and the source-receiver vector varied through 180°. The resulting thicknesses were compared with the thickness of the monostatic case and the resulting ΔRL in decibels was

plotted against the variable angle, A (Figures 9 and 10). From this plot, it can be concluded that the ΔRL resulting from the change of volume thickness can be related directly to the bistatic angle (Table 2). However, higher values of the bistatic angle, C will exist only in ellipses of higher values of eccentricity. Other results of this analysis are:

(1) As the eccentricity approaches zero (the ellipse approaches a circle), the thickness approaches the monostatic equivalent. The greatest ΔRL from the monostatic occurs at the higher eccentricity values.

(2) Small values of effective horizontal beamwidths, 2α, result in small ($<$1dB) bistatic adjustments. For values of the bistatic angle increasing beyond about 150°, the adjustment tends to grow rapidly until it approaches infinity. Errors inherent in the geometry also increase beyond this value.

For specific situations or for values of the bistatic angle not found in Table 2, Figures 9 and 10 may be used to derive a value of ΔRL. This can be combined with the specific bistatic adjustment to give the total variation of the bistatic reverberation level from the monostatic level. Or, the total contribution of the bistatic volume to the reverberation level may be calculated directly from the expressions of Chapter III if the specific parameters of the engagement are known.

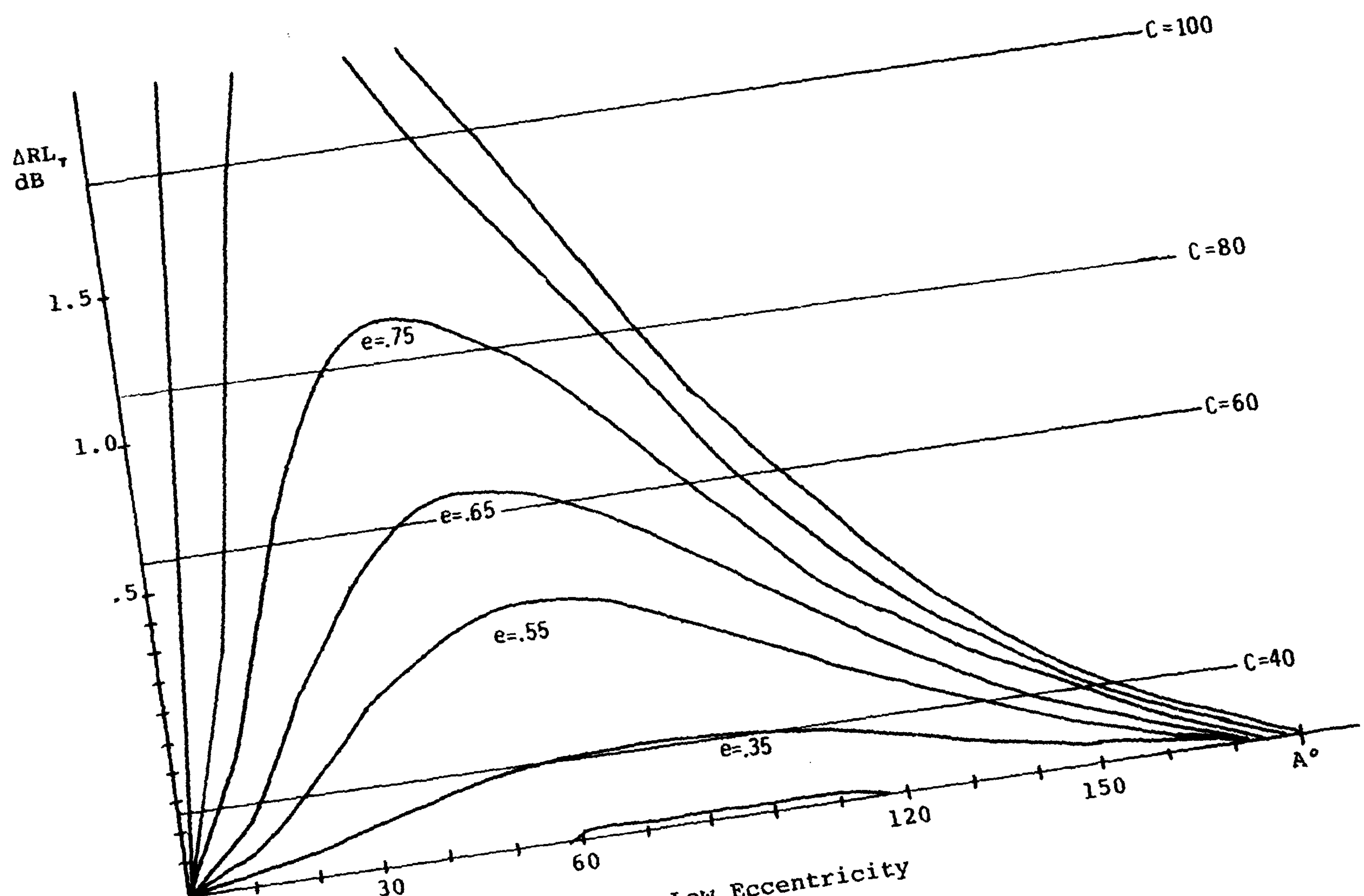

Figure 9. ΔRL$_\tau$ - Low Eccentricity

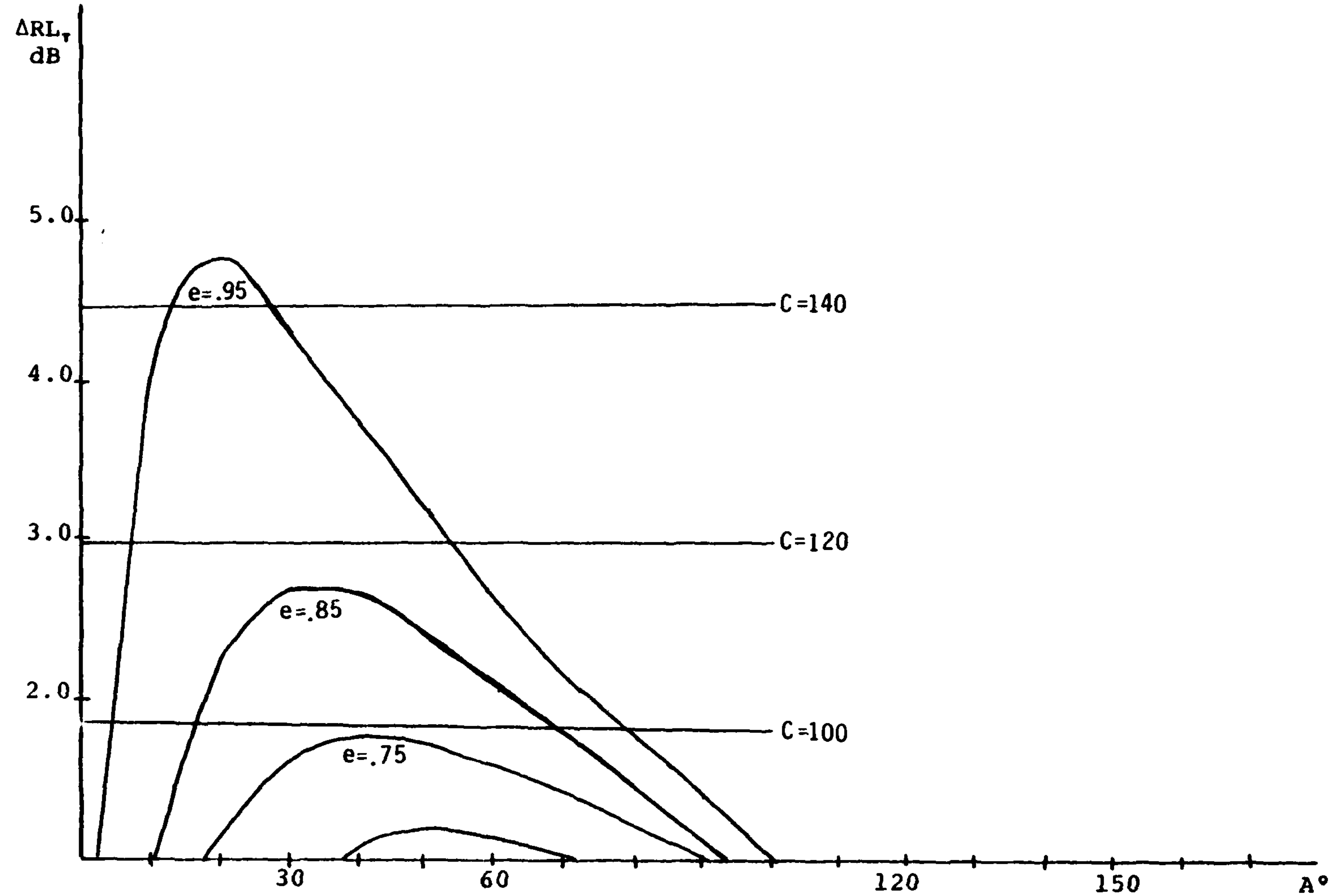

Figure 10. ΔRL_T - High Eccentricity

TABLE 2

ΔRL - BISTATIC ANGLE

C^{o}	ΔRL_{T} dB
10	.02
20	.07
30	.15
40	.27
50	.45
60	.62
70	.89
80	1.17
90	1.48
100	1.89
110	2.33
120	2.95
130	3.65
140	4.47
150	5.61
160	7.03
170	8.52

TABLE 3

BISTATIC ADJUSTMENT

$\alpha=1°$

C°	Bistatic Adjustment	10 log BA
0	1.000	0
10	1.0038	.017
20	1.0154	.067
30	1.0353	.151
40	1.0642	.270
50	1.1035	.428
60	1.1549	.625
70	1.2210	.867
80	1.3058	1.159
90	1.4149	1.507
100	1.5567	1.922
110	1.7451	2.418
120	2.0027	3.0163
130	2.3712	3.750
140	2.9339	4.675
150	3.8884	5.900
160	5.8445	7.668
170	12.1945	10.8616
180		--

TABLE 3 (cont'd)

BISTATIC ADJUSTMENT

$\alpha=5°$

C°	Bistatic Adjustment	10 log BA
0	1.000	0
10	1.0039	.017
20	1.0158	.068
30	1.0361	.154
40	1.0658	.277
50	1.1061	.438
60	1.1591	.641
70	1.2277	.891
80	1.3160	1.193
90	1.4306	1.555
100	1.5814	1.991
110	1.7851	2.5166
120	2.0709	3.162
130	2.4969	3.974
140	3.1968	5.047
150	4.5755	6.604
160	8.7990	9.444
170	1.96×10^{12}	122.9389
180		--

TABLE 3 (cont'd)

BISTATIC ADJUSTMENT

$\alpha=10°$

C°	Bistatic Adjustment	10 log BA
0	1.000	0
10	1.0042	.018
20	1.0169	.073
30	1.0388	.165
40	1.0708	.297
50	1.1147	.471
60	1.1729	.693
70	1.2492	.966
80	1.3495	1.302
90	1.4828	1.711
100	1.6648	2.214
110	1.9235	2.841
120	2.3164	3.648
130	2.9824	4.746
140	4.3671	6.402
150	9.0507	9.567
160	2.2764×10^{13}	133.57
170	--	--
180	--	--

As a general rule, the reverberation level will increase with increasing bistatic angle, approaching an infinite value when the bistatic angle is sufficiently large. Also, reverberant volumes may be generated for sonic beams that are other than right-circular conics providing that the cross-sectional area of the particular beam may be determined. The volume would then be the product of this area with the thickness and the adjustment.

To utilize Figures 9 and 10 and Table 3:

(1) Determine eccentricity

$$e = \frac{\text{Range Between Source and Receiver}}{\text{Total Range}}$$

(2) Determine angles A and α

(3) Enter Figure 9 with e and A to determine ΔRL_T resulting from volume thickness variation with respect to the bistatic geometry. Bistatic angle C may also be determined from these figures.

(4) Enter Table 3 with e and C to determine RL_G resulting from geometric effects.

(5) Sum of ΔRL_T and ΔRL_G is total change in monostatic reverberation level resulting from the bistatic configuration.

EXAMPLE:

Given: Total range = ct = $(1500\,^m/s)(4.5s)$ = 6750m; Range between source and receiver = 4000m; angle A = 40°, angle $\alpha = 5^\circ$

(1) $e = (4000) \div (6750) = .593$

(2) From Figure 9, $\Delta RL_{\tau} = .4$ dB, angle C = 55°

(3) From Table 3, $\Delta RL_{\theta} = .53$

(4) Total $\Delta RL = .4 + .53 = .93$ dB above the monostatic RL.

The potential advantages of a bistatic echo-ranging system may easily outweigh the disadvantages inherent in its physical employment if the theoretical properties of such a system are considered. Unfortunately, in the absence of contemporary experimentation and evaluation, the actual significance of these features is unknown. Possible advantages gained from employment of a bistatic system include the countering of some submarine acoustic treatments and evasive maneuvers while at the same time increasing the rate of data accumulation by the operator, thereby improving classification rates. The possibility of generating a weapons-firing solution without alerting the target is a tactical advantage as is the fact that the monostatic capabilities of either source or receiver are not affected. In fact, a potentially effective tactic that would increase the confusion of the target skipper is the random switching back and forth, between the bistatic elements, of the roles of source and receiver.

Physically, the bistatic geometry has its greatest effect on transmission loss, reverberation level, and target strength. Transmission loss can be calculated independent of the

particular geometry but reverberation and target strength are orientation dependent. By applying the properties of the ellipse to the bistatic geometry, a value specific to the situation may be obtained for the volume of reverberants. This volume is dependent upon the beam patterns of the components, the sourc-target and target-receiver ranges, the duration of the acoustic pulse, and the bistatic angle between the beams of the source and the receiver. Application of further features of propagation will allow the development of applicable surface reverberation areas. These values can then be utilized to generate volume and surface reverberation levels.

Target strength is presumed to have bistatic values different from those obtained by monostatic methods. Though the specific effects of source-receiver separation on target strength are not known precisely, theory based on radar applications suggests that bistatic values may be determined by manipulating monostatic strengths with respect to the bistatic angle.

The usefulness of bistatic echo-ranging has yet to be determined with respect to contemporary equipment, tactics, threats, and employment. Questions concerning future utility which must be answered include: (1) applicability of bistatic echo-ranging to specific existing platforms and systems, (2) physical features of such employment and their potential; especially attainable ranges, optimum separation

of components, and accuracy, (3) development of specific tactics involving these features, (4) feasibility of a multi-static scenario, utilizing one source and more than one receiver and, (5) cost-effectiveness of such a system with respect to physical assets such as equipment and communication channels. Only continued research and testing will provide the answers to these questions and thereby help evaluate the importance of bistatic echo-ranging. With the potential benefits of such a system, this determination should be made.

APPENDIX A

PROPERTIES OF THE ELLIPSE

An ellipse is the set of points in a plane whose distances from two given points in the plane have a constant sum. This geometric figure can be mechanically constructed in a number of ways. One example is by connecting a string of fixed length to two nails. If the string is kept taut by a pencil point which moves in the full range allowed, the pencil will describe an ellipse. Figure 11 identifies some of the major features of the ellipse. The foci are the two fixed points F and F'. The variable segment lengths $\overline{FP}$ and $\overline{F'P}$ are called focal radii. The sum of the focal radii at any given point P is of constant value, here set equal to the arbitrary value, 2a, also called the major axis. The points A and A' are called the vertices: Note that $\overline{A'A} = 2a$. The line through the foci is the transverse axis and the perpendicular bisector of segment $\overline{F'F}$ is the conjugate axis. The ellipse is symmetric about both the transverse and conjugate axes. The segment of the conjugate axis that connects the two points of intersection with the ellipse (points B and B'), is the minor axis. By definition, $\overline{BB'} = 2b$. Eccentricity refers to the degree of 'flatness' of the ellipse. Eccentricity, e, may be determined a number of ways but essentially, it is the ratio of the distance from the

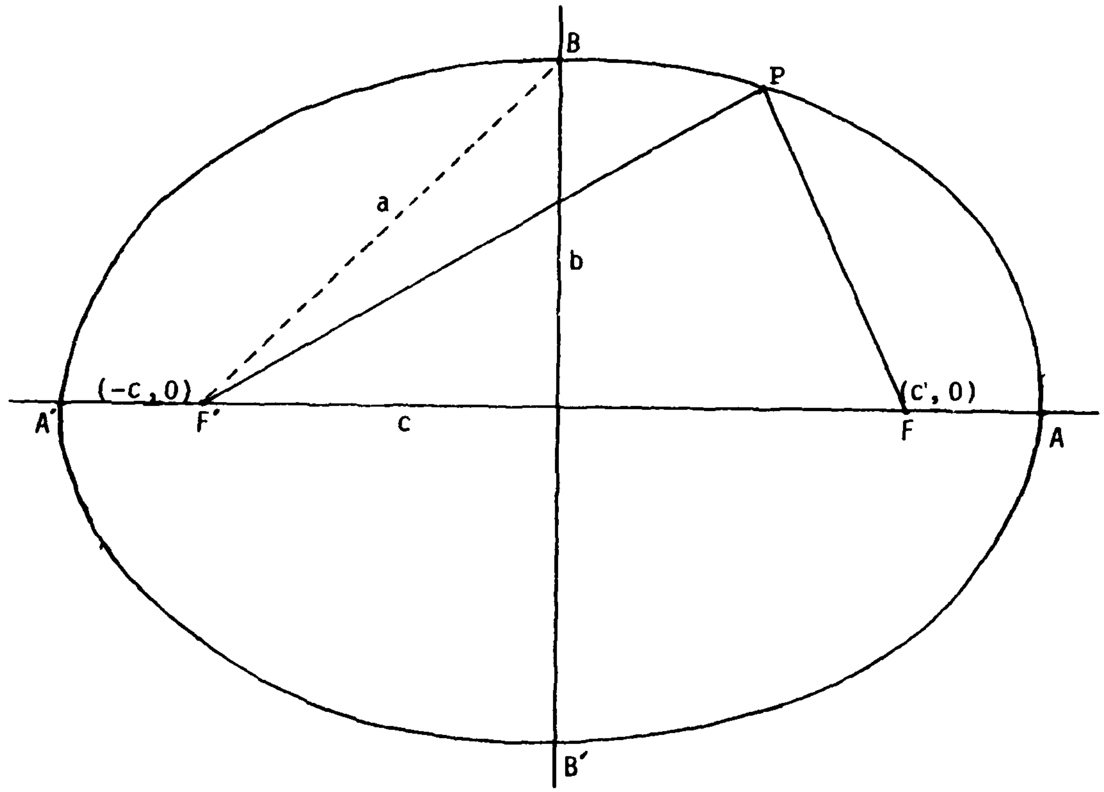

Figure 11. Ellipse

intersection of the axes to a focus, c, and the semi-major axis, a:

$$e = \frac{c}{a}.$$

If the eccentricity value approaches one, the ellipse approaches a straight line; if the eccentricity approaches zero, the ellipse approaches a circle.

If an ellipse were drawn with its center at the center of a set of orthogonal coordinate axes and with its axes overlaying the coordinate axes, then the equation for the ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Likewise, the length of the focal radii could be determined from

$$\overline{F'P} = a + ex \text{ and } \overline{FP} = a-ex.$$

A characteristic of the ellipse particularly useful in the development of the reverberant volume is the reflection property. This rule states that the focal radii, drawn to a point P on the ellipse, make congruent angles with the tangent to the ellipse at point P. Additionally, the area contained by the locus of points P, and therefore the ellipse, is

$$A = \Pi ab.$$

To summarize the main features of the ellipse:

a = Semi-major axis: Distance from the center to a major vertex; half the sum of the focal radii; distance from a focus to a minor vertex.

b = Semi-minor axis: Distance from the center to a minor vertex

c = Distance from the center to a focus.

APPENDIX B

DETERMINATION OF MAJOR AND MINOR AXES OF THE TARGET ELLIPSE

It is necessary to calculate expressions for the major and minor axes of the target ellipse in terms of the assumptions, mathematical properties, and known quantities. Figure 12 depicts the preliminary geometry involved in this development. The cone of sound transmission of the smallest dimensions at the target has vertex V and axis $\overline{VV'}$. The intersecting plane contains the line $\overline{TT'}$. The line segment $\overline{WP}$ represents the axis of the cone of transmission of the greatest dimensions. The focal radii of the range ellipse at the target's position P are represented by the segments $\overline{VP}$ and $\overline{WP}$. The reflection property of the ellipse results in the equality of the two angles

$$VPT' = WPT.$$

Further application of the geometry of vertical angles yields

$$W'PT = WPT = VPT' = V'PT.$$

In the bistatic situation, expanding Figure 12 and Figure 13, side c and angle B are known as well as either side a or side b. Angle C is unknown. Depending on the known elements, angle C can be determined by either the Law of Sines or Cosines. If two sides and the included angle are known,

$$S_3 = (S_1{}^2+S_2{}^2 - 2S_1S_2 \cos A_1)^{\frac{1}{2}}$$

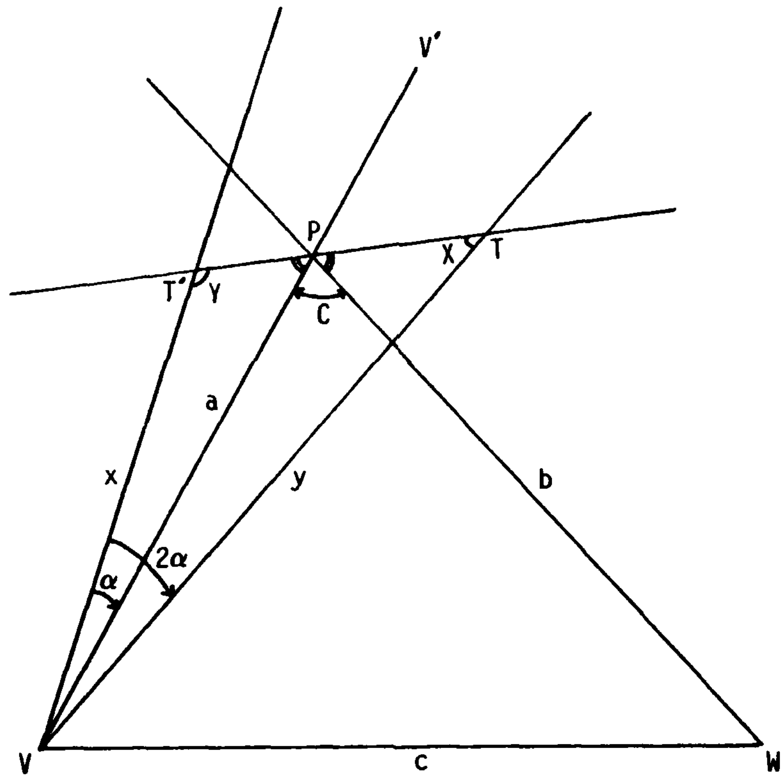

Figure 12. Bistatic Geometric Argument

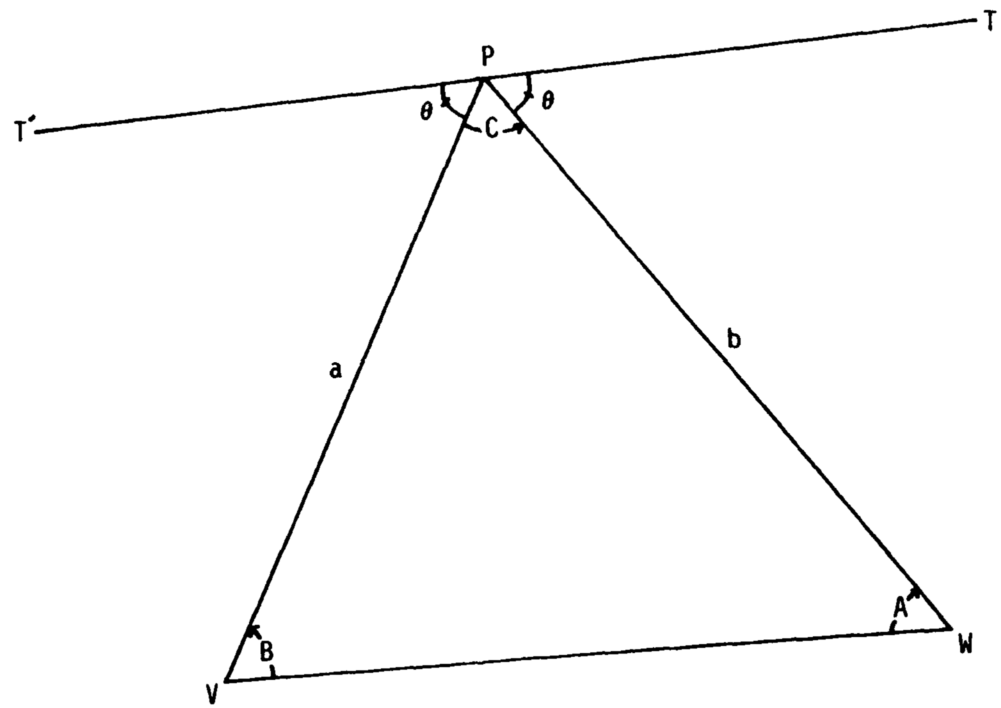

Figure 13. Bistatic Triangle

which allows further determination by the Law of Sines of angle C. If two sides and an adjacent angle are known,

$$A_3 = \sin^{-1}[\frac{S_2}{S_1} \sin A_2]$$

again an application of the Law of Sines. The angle Θ is then equal to,

$$\Theta = 1/2\ (180-C).$$

From these results, angles X and Y may be easily determined. Application of trigonometric properties would then allow for the determination of the major and minor axes in terms of angles α, X, Y and ranges a, c, and x. The calculations inherent in the use of these terms, while workable, are complex and require several intermediate solutions. The minor axis is particularly difficult to determine because this axis, though a function of the conic radius, varies with respect to the location of the mid-point of the major axis.

Another geometric approach results in comparable values for the major and minor axes but in a more direct and concise manner [Ref. 6]. For this reason, this approach will be further expanded and used in this development. The geometry previously established is still valid and is utilized in the second technique.

A normal to the plane intersecting the cone of sound, if the normal is drawn from the target point, will form an angle of $^{C}/2$ with the axis of the cone. If the cone is aligned in a coordinate system (Figure 15), so that the vertex (source

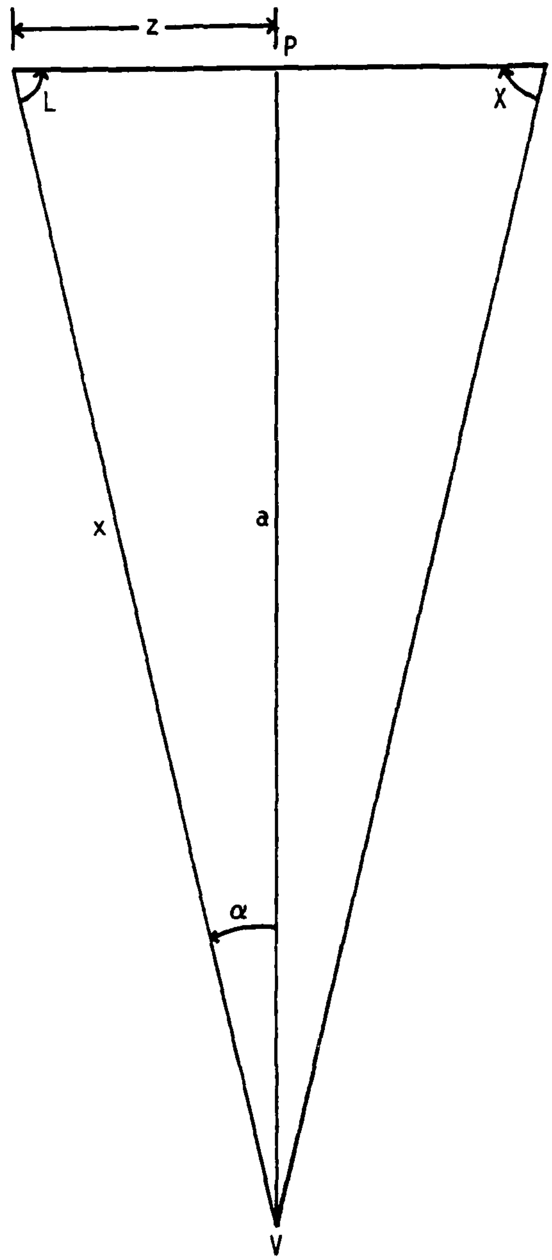

Figure 14. Right Circular Cone

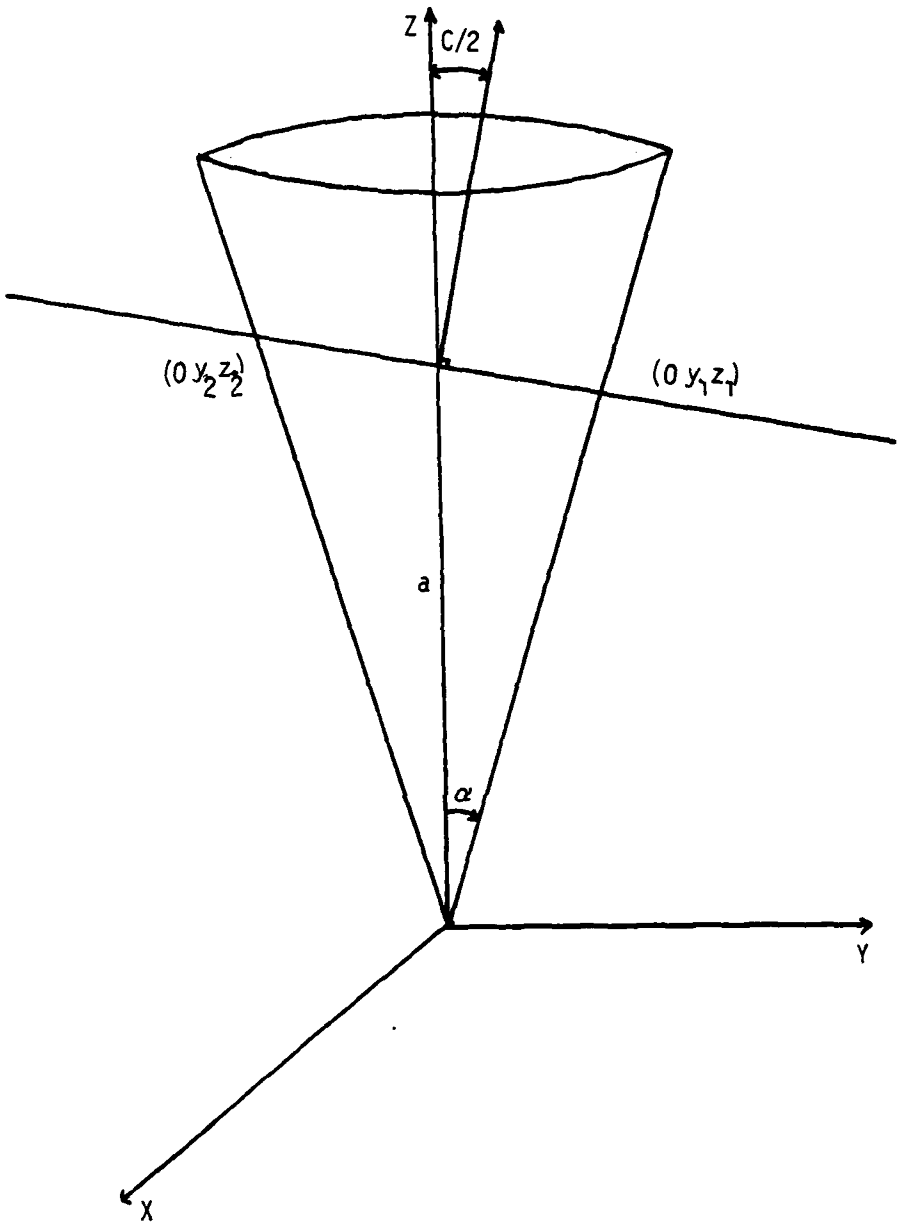

Figure 15. Conic Geometry

or receiver), is at the origin and the conic axis lies on the z-axis of the system, the expression for the cone will be

$$x^2+y^2 = z^2\tan^2\alpha.$$

The intersecting plane is perpendicular to the Y-z plane. Since the target lies on the conic axis, it can be represented as point (0,0,a). Solution of the equation for the plane at the target point results in a value of z that is common to both the cone and the plane,

$$z = a-y\tan(C/2).$$

This z value, expressed in the equation of the cone, will generate an equation for the target ellipse,

$$x^2+y^2 = [a-y\tan(C/2)]^2\tan^2\alpha.$$

The geometry was selected so that x = 0 along the major axis. Therefore, solution of the target ellipse equation may be accomplished in terms of Y and z, resulting in the values of the major axis end-points,

$$(o,y_1,z_1)=\left(0,\frac{a\tan\alpha}{1+\tan\alpha\tan(C/2)},\frac{a}{1+\tan\alpha\tan(C/2)}\right) \text{ and}$$

$$(0,y_2,z_2) = \left(0,\frac{-a\tan\alpha}{1-\tan\alpha\tan(C/2)},\frac{a}{1-\tan\alpha\tan(C/2)}\right)$$

The distance between these points, which is the length of the major axis, may then be solved using the equation

$$2a = [(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2]^{\frac{1}{2}}.$$

The length of the semi-major axis is

$$\frac{M}{2} = \frac{a\tan\alpha\sec(C/2)}{1-\tan^2\alpha\tan^2(C/2)} .$$

The minor axis and the major axis intersect at one point, their common mid-point, which is also the center of the ellipse. The minor axis, in this arbitrary geometry, is constant in Y and Z and varies over X. Solution of the mid-point of the major axis results in the Y and Z values for the minor axis. Since the minor axis of the target ellipse extends across the width of the cone, the axis end-points must also lie on the cone. Solving the expression for the cone in terms of the Y and Z values of the minor axis results in a semi-minor axis length of

$$\frac{N}{2} = \frac{a\tan\alpha}{[1-\tan^2\alpha\tan^2(C/2)]^{1/2}}.$$

Since the area of an ellipse is the product of pi and the semi-axes, the area of the target ellipse becomes,

$$A_T = \frac{\pi a^2\tan\alpha\sec(C/2)}{[1-\tan^2\alpha\tan^2(C/2)]^{3/2}}.$$

In the monostatic case, (Figure 14), the radius of the right-circular cone at the target point is

$$Z = a \tan \alpha.$$

So, in terms of the equivalent monostatic cross-sectional area, the area of the target ellipse becomes

$$A_T = \frac{\pi Z^2\sec(C/2)}{[1-\tan^2\alpha\tan^2(C/2)]^{3/2}}.$$

APPENDIX C

THICKNESS OF REVERBERATION VOLUME

The thickness of the reverberation volume is the dimension over which the area of the target ellipse is multiplied to produce the volume measurement. This thickness represents the space between the perimeters of two ellipses each with the same value for c for different values of the semi-major axis. The result of this inequality is non-similar ellipses; they are of different eccentricities and there is no correspondence between their elements. For this reason, an approximation of the distance between ellipses is most practical for this development.

The perpendicular to the tangent to the range ellipse at the target point P will intersect a similar, but not equal, tangent drawn to the outer ellipse. The length of this perpendicular between the intersected ellipses will provide an accurate approximation of the thickness of the reverberating volume. For targets at either major or minor vertices, the thickness will be $a_{outer} - a_{inner}$ or $b_{outer} - b_{inner}$, respectively. This thickness will be greater for targets on the minor vertices than for targets on the major vertices. For targets between the vertices, the thickness will vary and must be determined using the perpendicular assumption.

The perpendicular distance can be determined by manipulation of the polar equations of the inner and outer ellipse,

$$r = \frac{a(1-e^2)}{1-e\cos\Theta}$$

The value r is the focal radius, and Θ is the angle between the line connecting foci and the focal radius. By solving for the focal radius of the inner and outer ellipse using the value for Θ that allows both radii to pass through the target point, the slant distance, ℓ, between ellipses may be determined (Figure 16),

$$\ell = \frac{(a_1-a_1e_1^{\,2})(1-e_2\cos\Theta)-(a_2-a_2e_2^{\,2})(1-e_1\cos\Theta)}{(1-e_1\cos\Theta-e_2\cos\Theta+e_1e_2\cos^2\Theta)}$$

The thickness of the monostatic reverberation volume, $^{c\tau}/2$, is incorporated in the values of semi-major axes of the inner and outer ellipse, a_2 and a_1. Since the perpendicular bisects the angle formed by the radii from the source and receiver, a right triangle may be generated with known angle $^{C}/2$, and hypotenuse ℓ. Application of trigonometry results in an expression for the approximate thickness, p,

$$p = \ell\cos{}^{C}/2$$

This approximation is valid when $^{c\tau}/2 << R_1+R_2$. The error associated with the perpendicular assumption is of small enough value that for the geometries considered in bistatic applications, it can be considered insignificant.

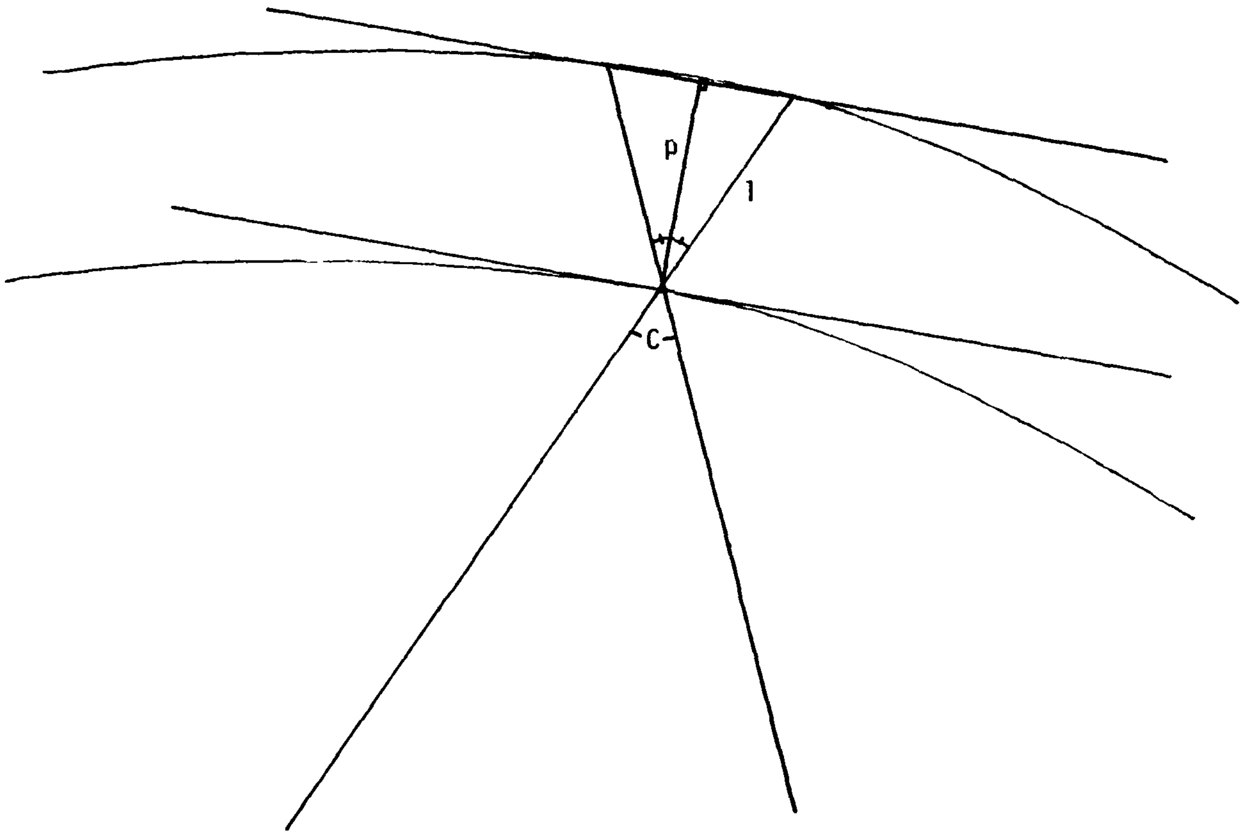

Figure 16. Volume Thickness

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